

PSYC214: Statistics Lecture 2 – One factor between-participants ANOVA – Part I

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One factor between-participants ANOVA



Agenda/Content for Lecture 2

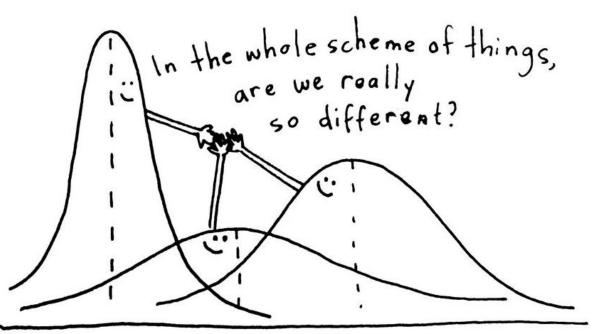
- Introduction to analysis of variance (ANOVA)
- Introduction to one factor betweenparticipants design
- Sources of variability in data
- Calculating within-group and betweengroup variances
- Degrees of Freedom
- Producing the F-statistic





Why conduct an analysis of variance?

- Compares means and variance
- Allows analysis of group differences for more than two groups
- Several means without inflating Type I error rate



Source: Questionpro

Dissertation!



ANOVA is a good weapon of choice!

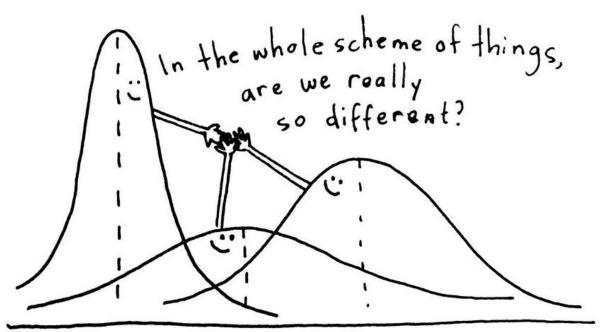






What do you need for a one factor between participants ANOVA?

- Three or more separate groups
- ONE categorical independent variable (i.e., one factor)
- One <u>continuous</u> dependent variable (outcome measure)



Source: Questionpro

Sources of variability in data



- 1. Treatment effects
- 2. Individual differences
- 3. Random (residual) errors



Within-group variability?



Between-group variability?

Sources of variability in data



- **1. Treatment effects**
- 2. Individual differences
- 3. Random (residual) errors



Within-group variability?



Between-group variability?

Treatment effects

- The effects of the independent variable
- This is what we want!
- We want people who are treated differently because of our intervention to behave differently



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Sources of variability in data



- 1. Treatment effects
- 2. Individual differences
- 3. Random (residual) errors

Individual differences



- Some individuals may be more proficient in memory recall
- Maybe some individuals have experience of similar tasks
- Some may have ignored instructions or had lower attention spans / motivation
- A control group can employ their own strategy, increasing the variability



Sources of variability in data



- 1. Treatment effects
- 2. Individual differences
- 3. Random (residual) errors

Random (residual) errors



- Ideally a participant would have a 'true level' at which they perform, which can always be measured accurately
- 1. Varying external conditions e.g., temperature, time of day
- 2. State of participant (e.g. tired?)
- 3. Experimenter's ability to measure accurately...



... Experimenter effects



- Experimenters need to minimise these, so not to obscure the treatment effect
- Spread data away from the true means – i.e., increase variability and standard errors
- Reduce confidence in our estimates and a randomly plucked sample



Within- and between- group variability



Within-group variability

The extent to which participants within a single group or population differ, despite receiving the same treatment



Within-group variability?

Between-group variability

The extent to which overall groups differ from one another (hopefully because of our treatment)



Within- and between- group variability



High between-group variability **No** within group-variability **No** between-group variability High within-group variability

Moderate between-group variability Moderate within-group variability

	Group A	Group B	Group C
	10	20	30
	10	20	30
	10	20	30
	10	20	30
	10	20	30
Mean	10	20	30
S	0	0	0

	Group A	Group B	Group C
	10	15	5
	25	20	25
	30	30	25
	35	40	45
	50	45	50
Mean	30	30	30
S	14.6	12.8	18.0

	Group A	Group B	Group C
	10	10	20
	10	20	20
	10	20	30
	20	20	30
	20	30	30
Mean	14	20	26
S	5.5	7.1	5.5



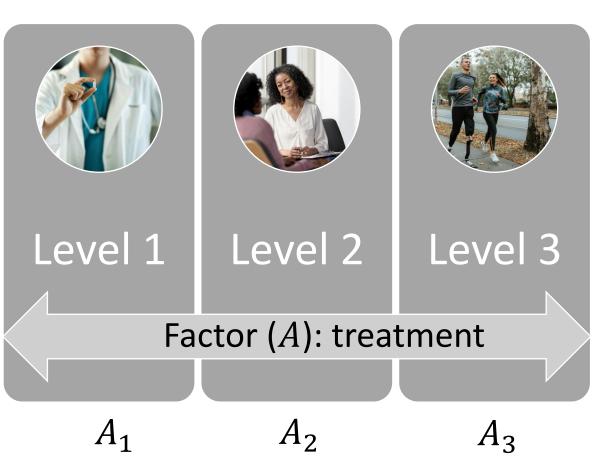
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Factors and levels (Example 1)

- Factor: treatment
- 3 levels
 - Medication
 - Counselling
 - Exercise

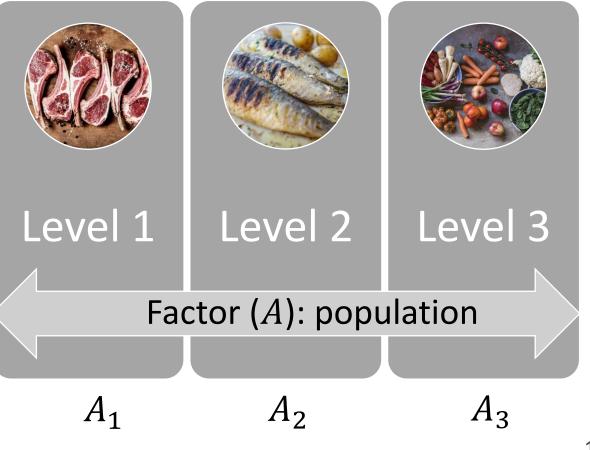




Factors and levels (Example 2)

- Factor: **population**
- 3 levels:
 - A₁ Meat eater
 - A₂ Pescatarian
 - A₃ Vegetarian

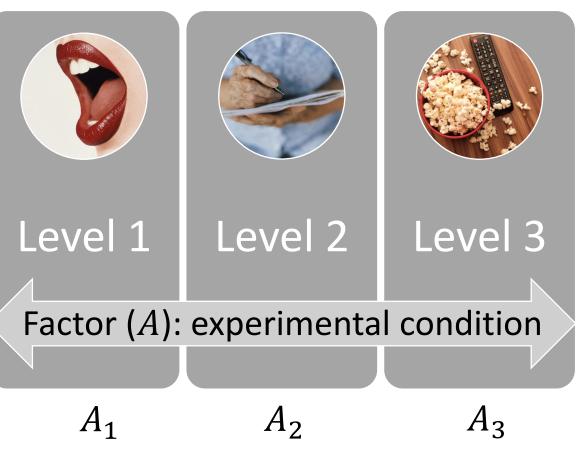




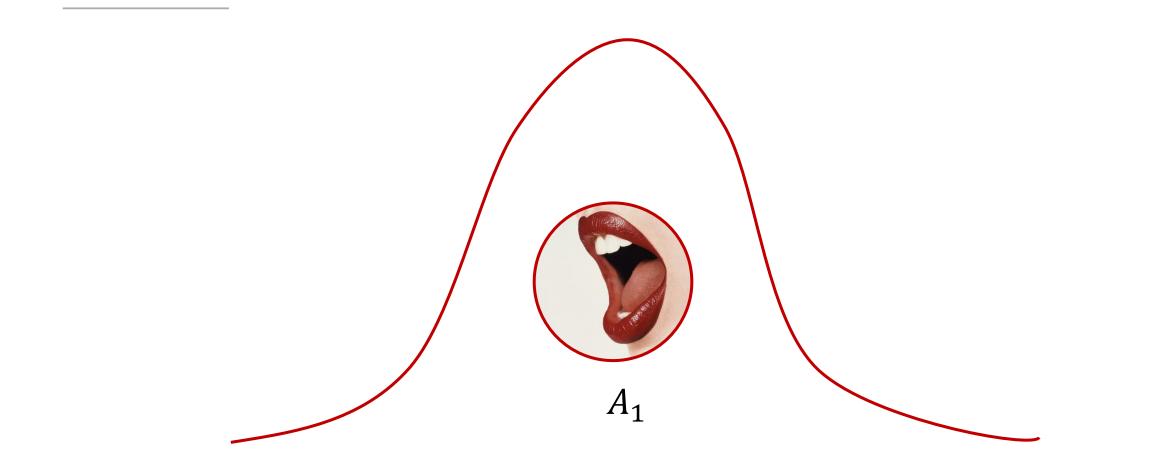


Factors and levels (Example 3)

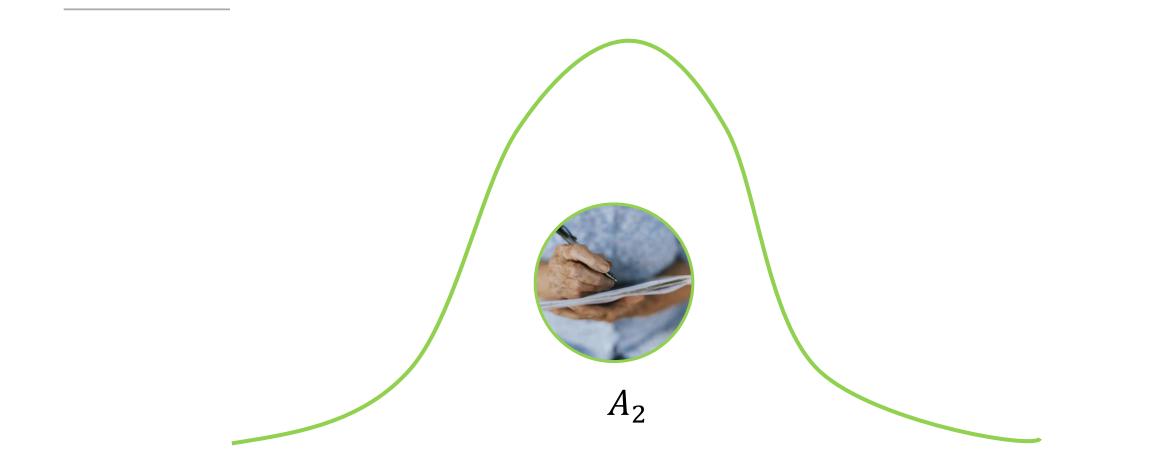
- Factor: experimental condition
- 3 levels:
 - A₁ Verbal negative feedback
 - A₂ Written negative feedback
 - A₃ Control (no feedback)



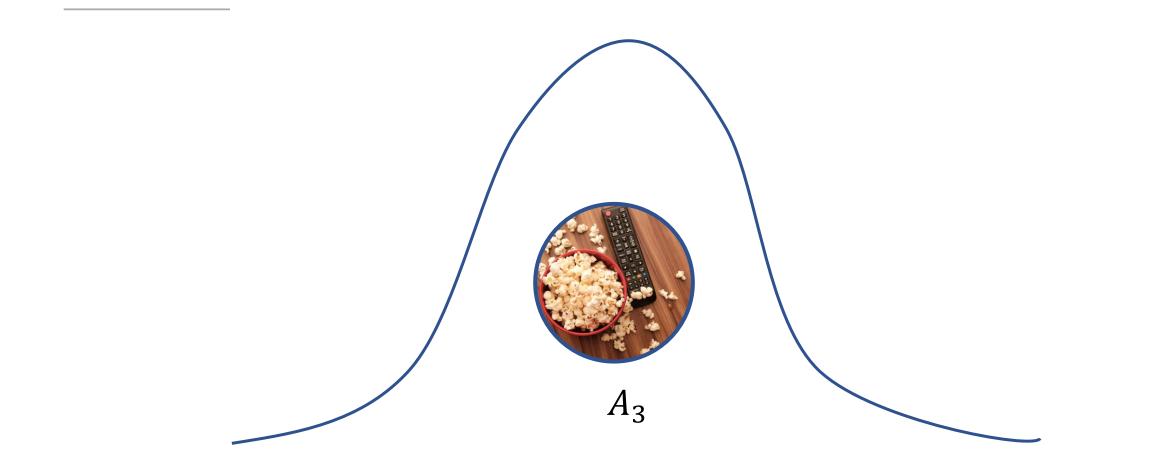




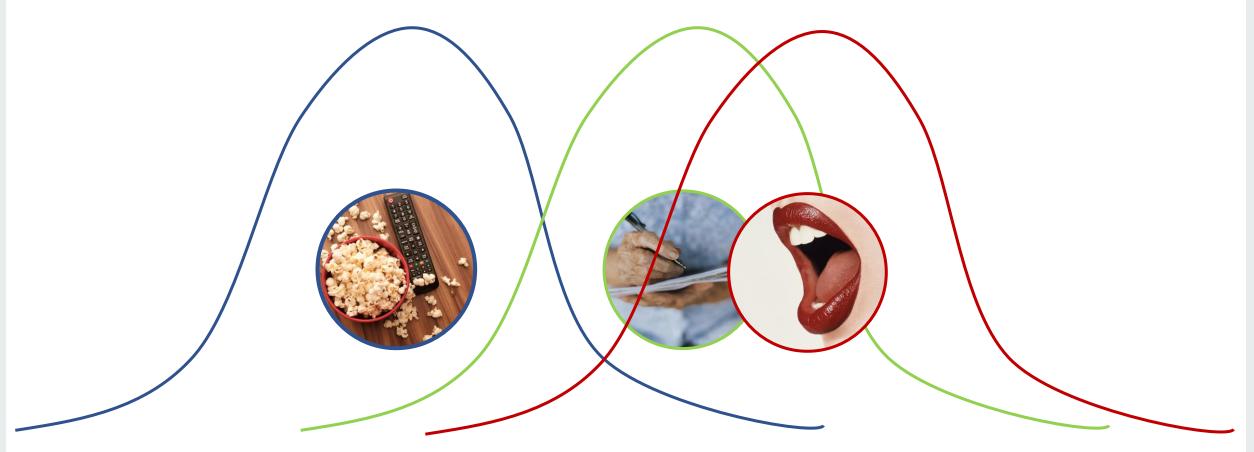




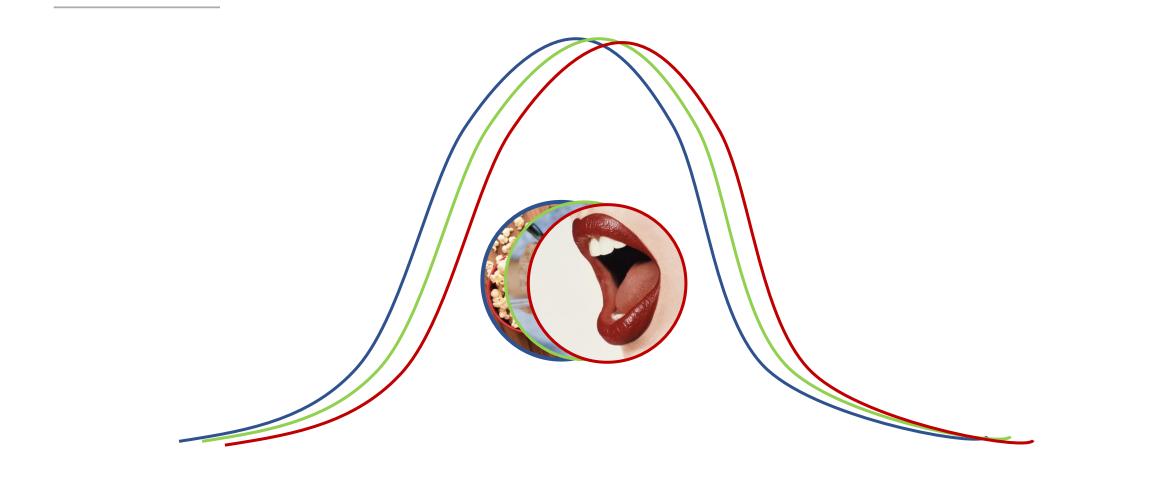










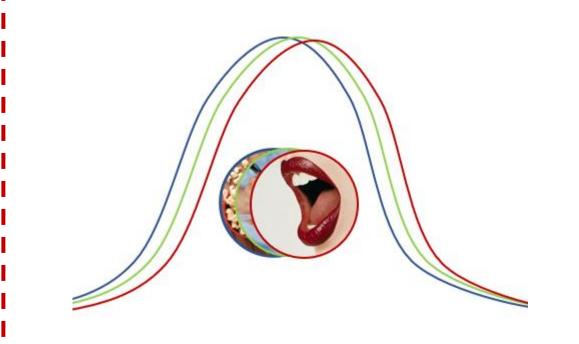


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Testing for differences

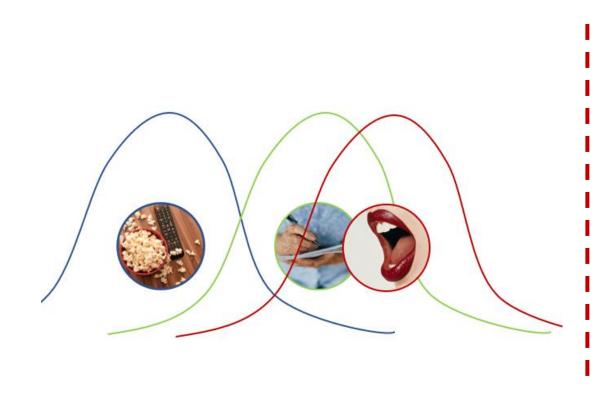
• Ho the Null Hypothesis

- Under H0, the samples come from the same population
- $\mu_1 = \mu_2 = \mu_3$ [No difference in the population means]
- Experimental effect = 0
- All differences are due to individual differences + random (residual) errors

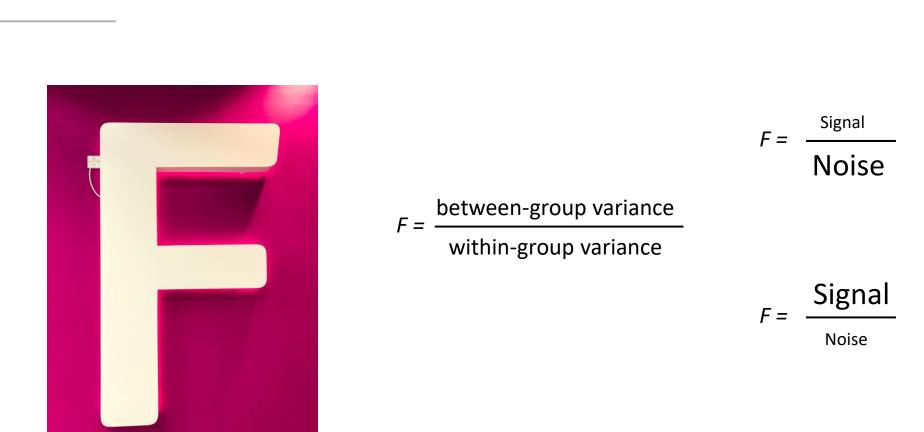


Testing for differences

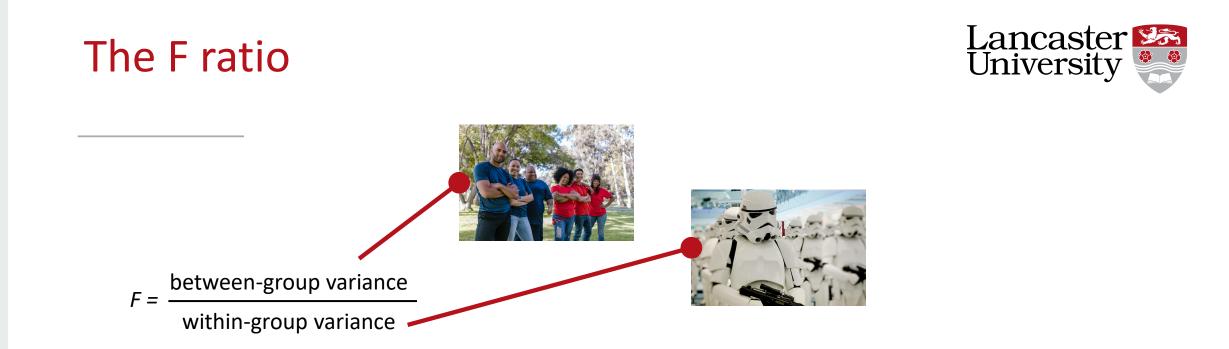


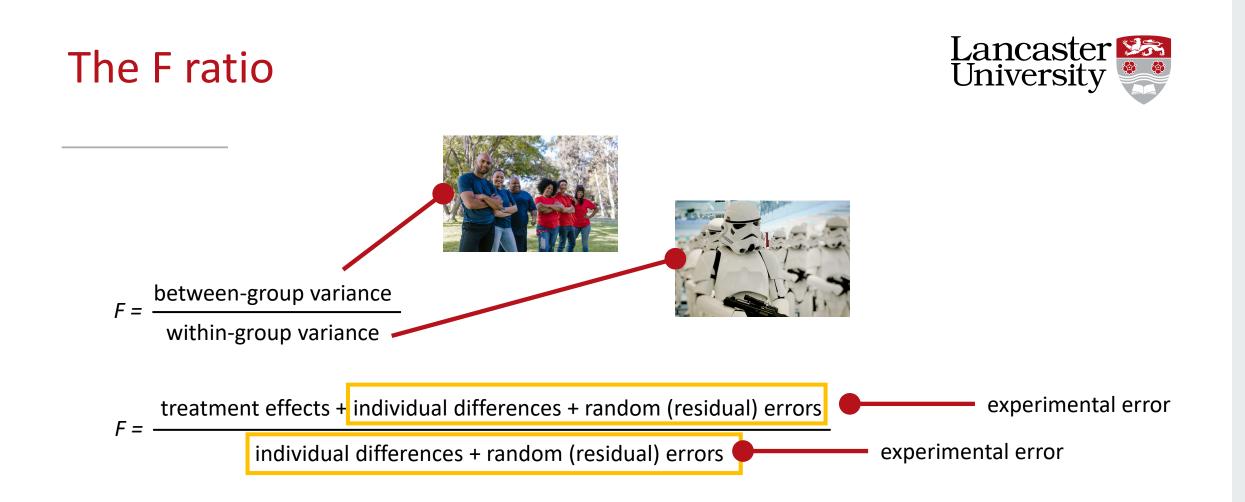


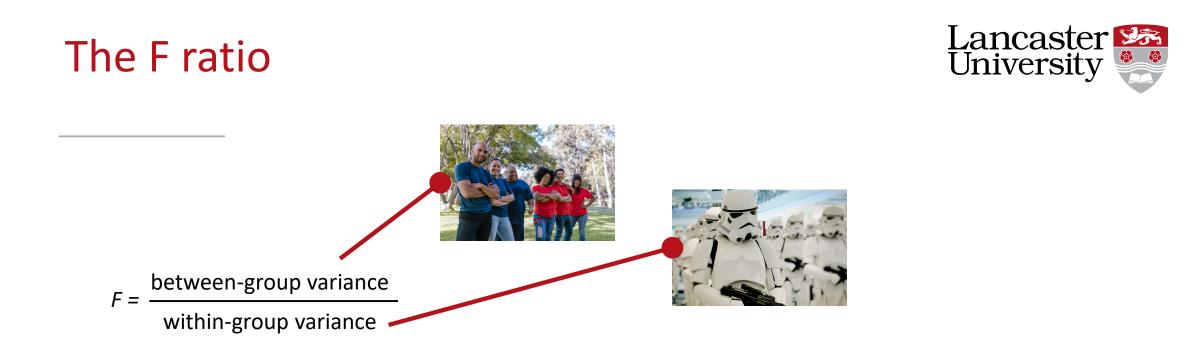
- H1 the Experimental Hypothesis
- Under H1, the samples come from the <u>different</u> populations.
- $\mu_1 \neq \mu_2 \neq \mu_3$ [Population means are different]
- Experimental effect $\neq 0$
- Differences are due to individual differences, random (residual) errors <u>AND</u> the experimental effect











individual differences + random (residual) errors

F = _____

experimental error





The more treatment effects are standing out away from experimental error – i.e., the larger the signal is from the noise, the larger in magnitude the F value. The larger the F, the less likely that differences in scores are caused by chance.



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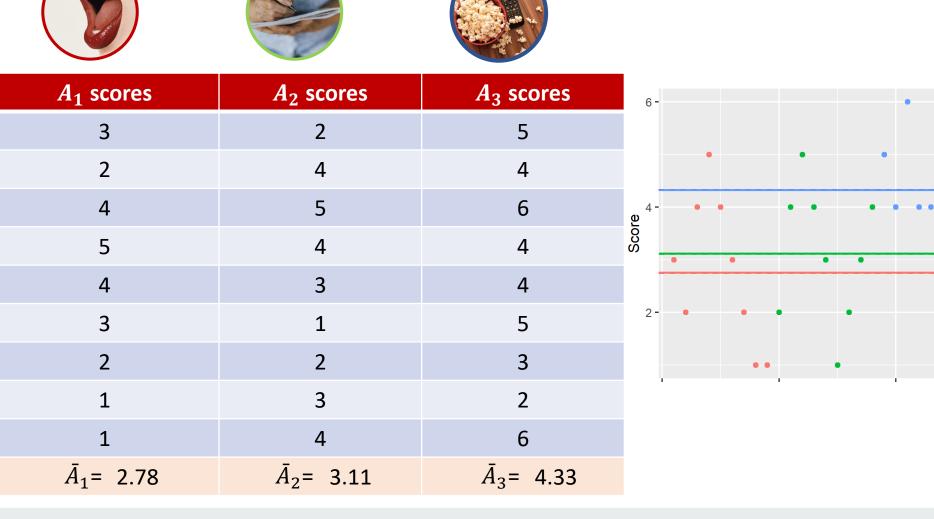
Calculating between-group variance





$F = \frac{\text{between-group variance}}{\text{within-group variance}}$

Mea	an (Ā)			Lancaster Standart University
	A ₁ scores	A ₂ scores	A ₃ scores	Total set of scores
	3	2	5	iotal set of scores
	2	4	4	
	4	5	6	– Å <i>Y</i>
	5	4	4	$\left[-\frac{1}{V} \right] $ $C \Lambda$
	4	3	4	A = -
	3	1	5	IV
	2	2	3	
	1	3	2	Number of scores
	1	4	6	Mean



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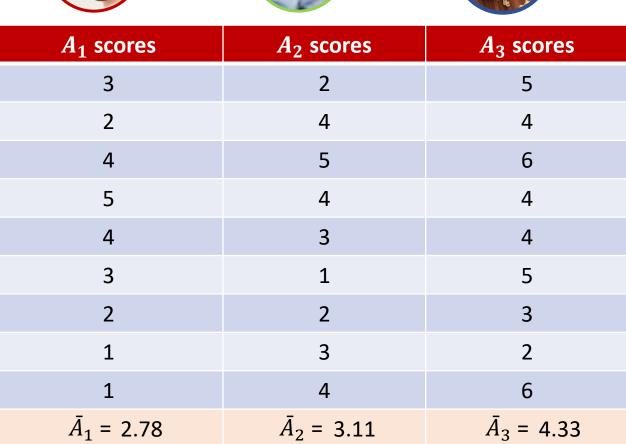


.

Grand Mean (\bar{Y})









$$\bar{Y} = \frac{\bar{A}_1 + \bar{A}_2 + \bar{A}_3 + \dots \bar{A}_k}{k}$$

$$\bar{Y} = The \ grand \ mean \ of \ averages$$

k = number of levels

$$\bar{Y} = \frac{2.78 + 3.11 + 4.33}{3}$$
$$\bar{Y} = 3.41$$

Grand Mean (\bar{Y})

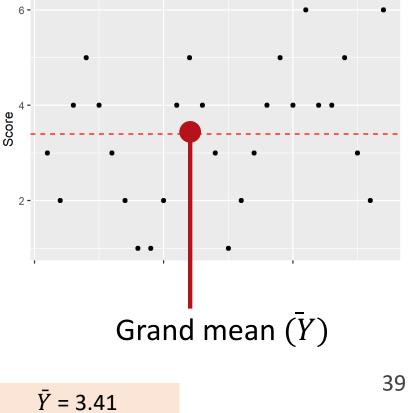






A ₁ scores	A ₂ scores	A ₃ scores	(
3	2	5							
2	4	4							
4	5	6	Score						
5	4	4							
4	3	4							
3	1	5							
2	2	3							
1	3	2							
1	4	6							
\bar{A}_1 = 2.78	<i>Ā</i> ₂ = 3.11	Ā₃= 4.33							



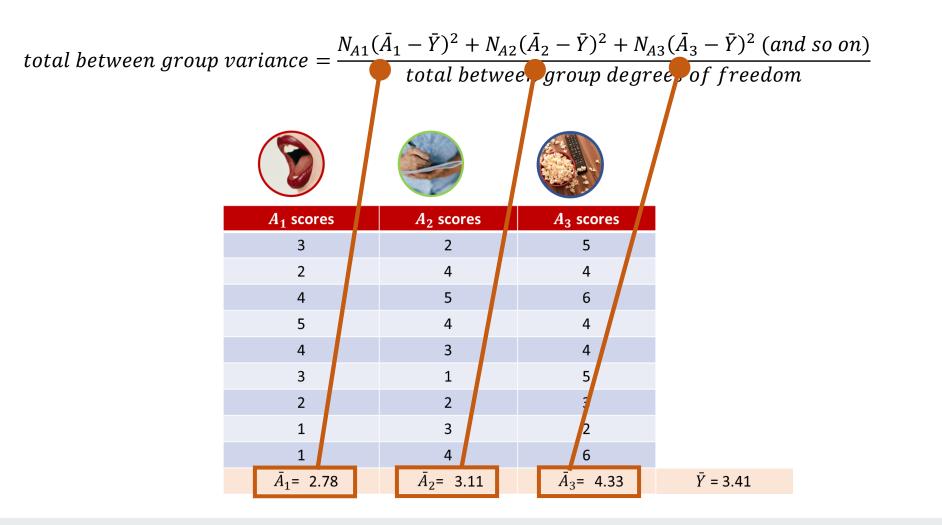




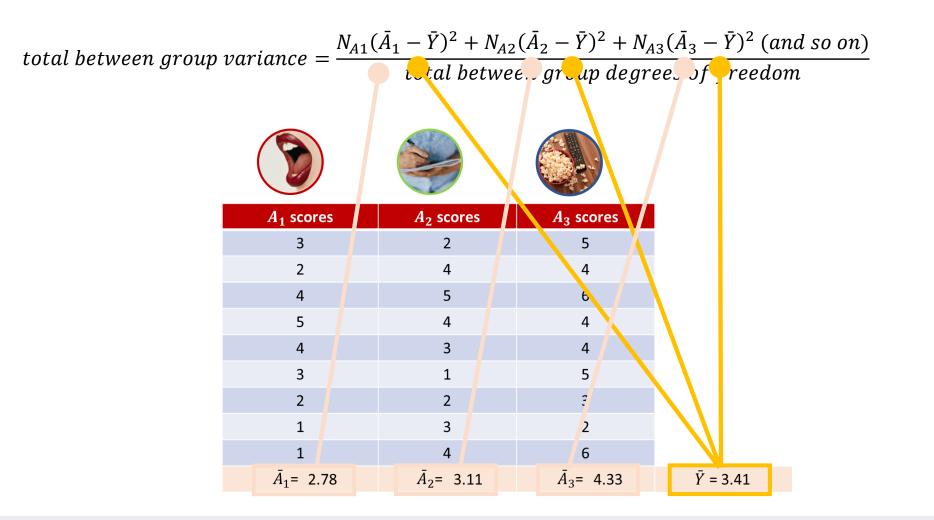
 $total \ between \ group \ variance = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2 \ (and \ so \ on)}{total \ between \ group \ degrees \ of \ freedom}$

A ₁ scores	A ₂ scores	A ₃ scores			
3	2	5			
2	4	4			
4	5	6			
5	4	4			
4	3	4			
3	1	5			
2	2	3			
1	3	2			
1	4	6			
\bar{A}_{1} = 2.78	\bar{A}_2 = 3.11	Ā₃= 4.33			

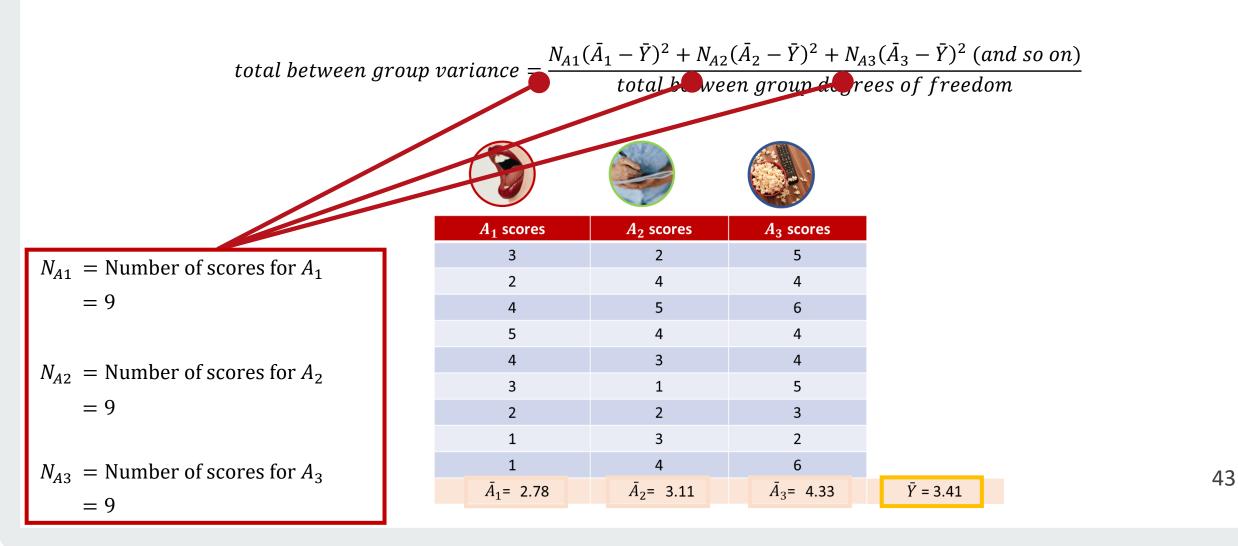












Degrees of freedom

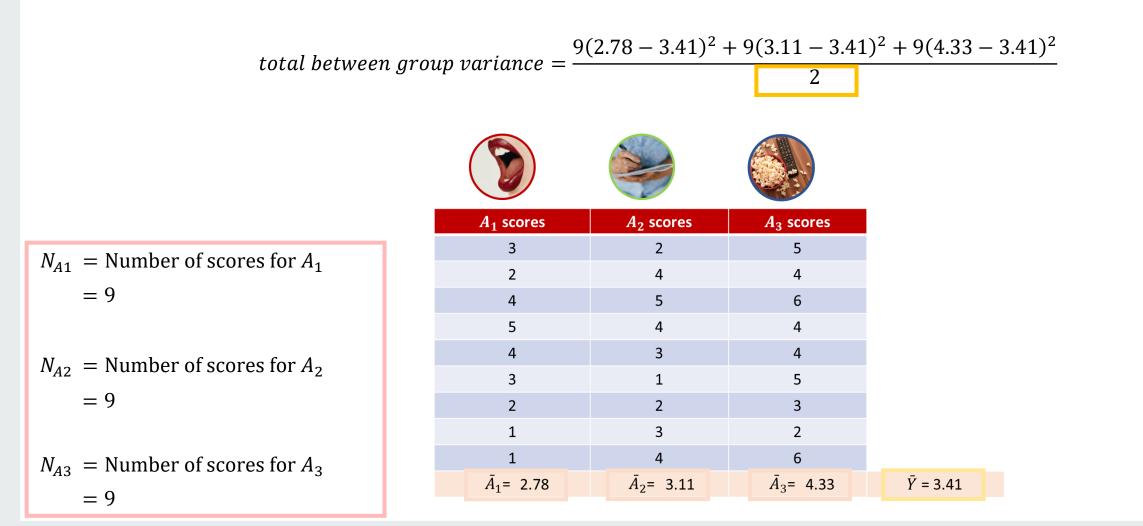
Between-groups degrees of freedom

- The total number of levels minus one
- For example, in our experiment we have three levels [verbal feedback, written feedback, control]
- The between-groups degree of freedom is there 3 levels – 1 = 2
- Between-groups df = 2

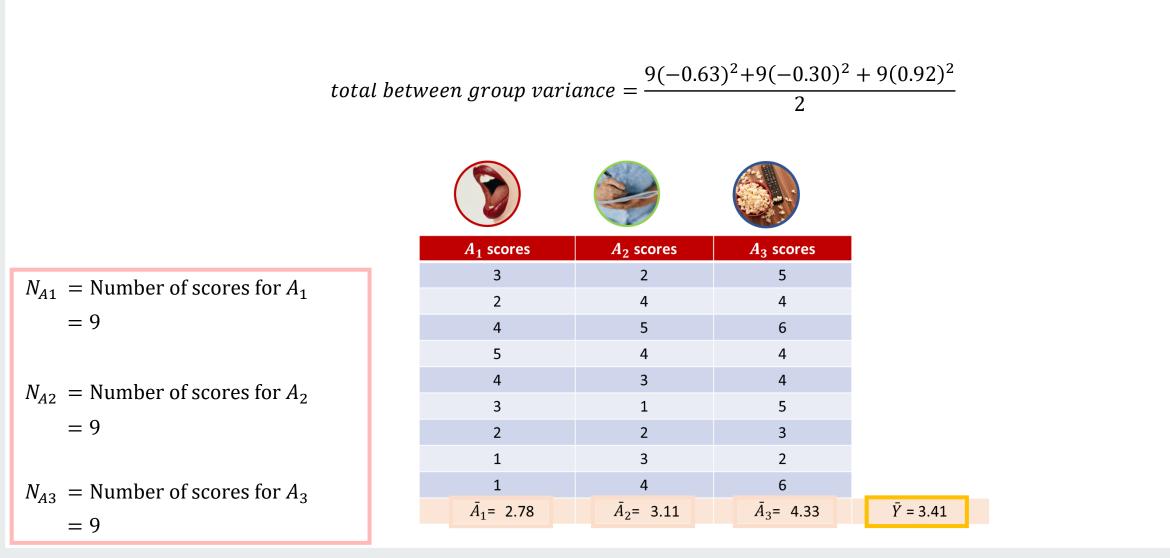




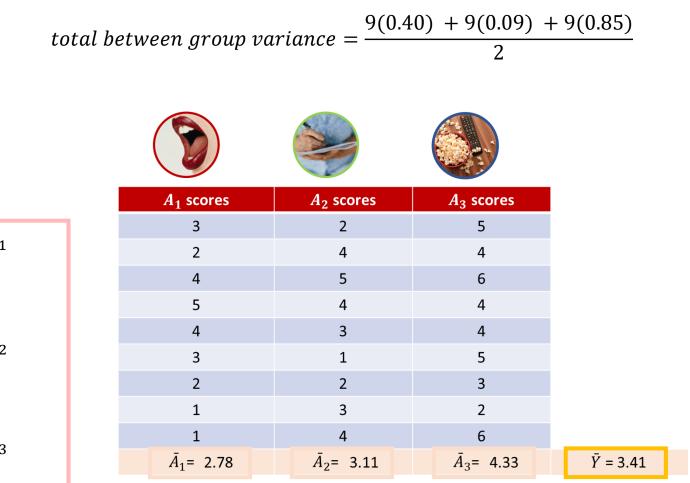










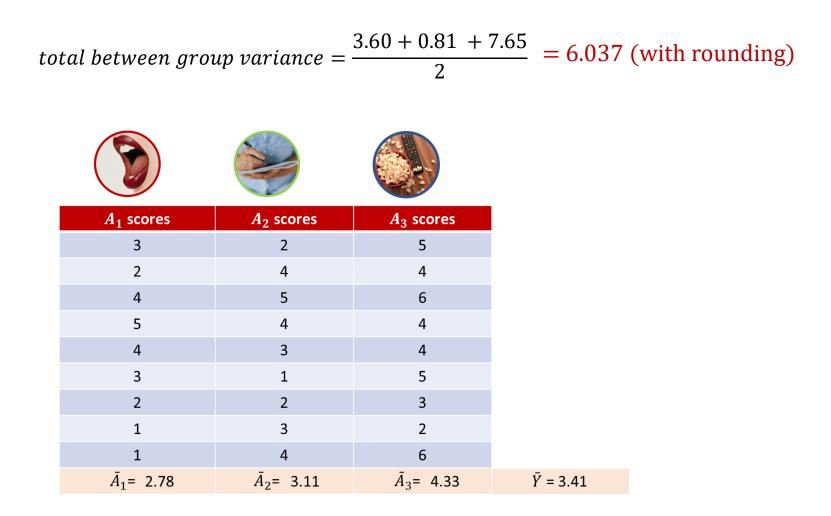


 N_{A1} = Number of scores for A_1 = 9

 N_{A2} = Number of scores for A_2 = 9

 N_{A3} = Number of scores for A_3 = 9





Calculating between-group variance





$F = \frac{\text{between-group variance}}{\text{within-group variance}}$

 $F = \frac{6.037}{\text{within-group variance}}$



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Up to now...





 $F = \frac{\text{between-group variance}}{\text{within-group variance}}$

 $F = \frac{6.037}{\text{within-group variance}}$

Calculating within-group variance





 $F = \frac{\text{between-group variance}}{\text{within-group variance}}$



 $total within group variance = \frac{SS \ level \ A_1 + SS \ level \ A_2 + SS \ level \ A_3 (and \ so \ on)}{total \ within \ group \ degrees \ of \ freedom}$

Mean

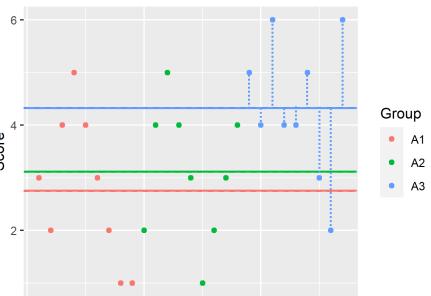




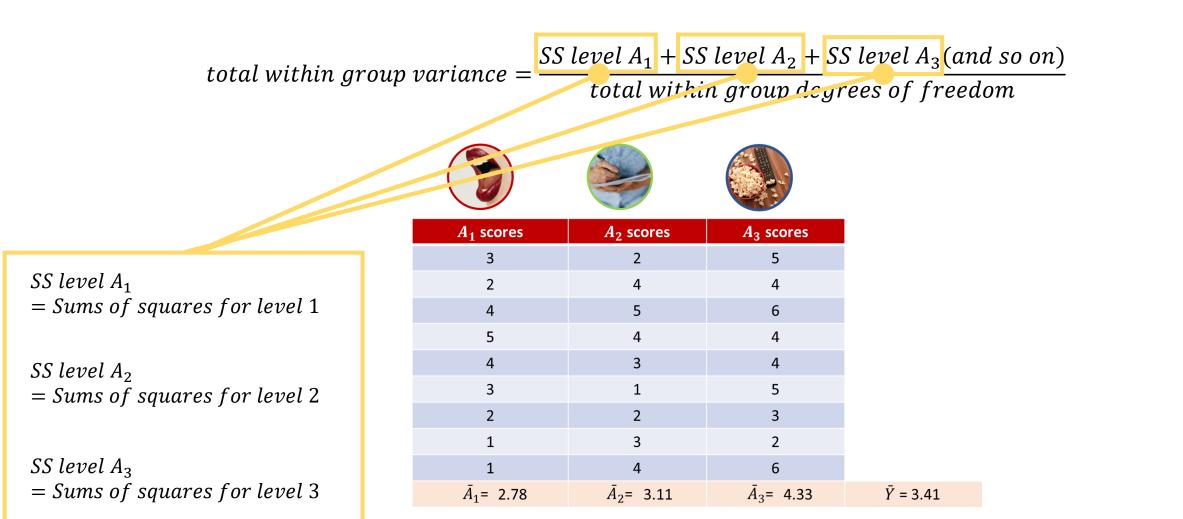




A ₁ scores	A ₂ scores	A ₃ scores	6
3	2	5	
2	4	4	
4	5	6	و 4 و
5	4	4	4 Score
4	3	4	
3	1	5	2
2	2	3	
1	3	2	
1	4	6	
\bar{A}_1 = 2.78	\bar{A}_2 = 3.11	\bar{A}_3 = 4.33	

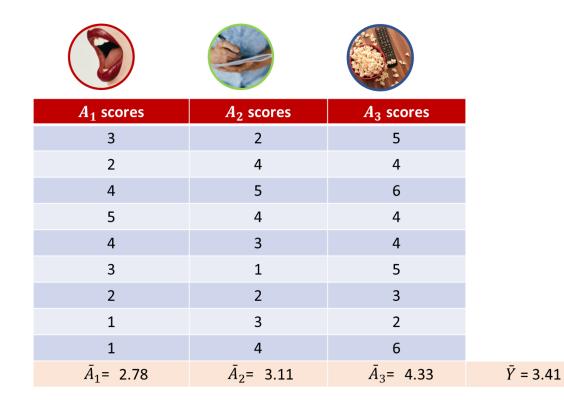




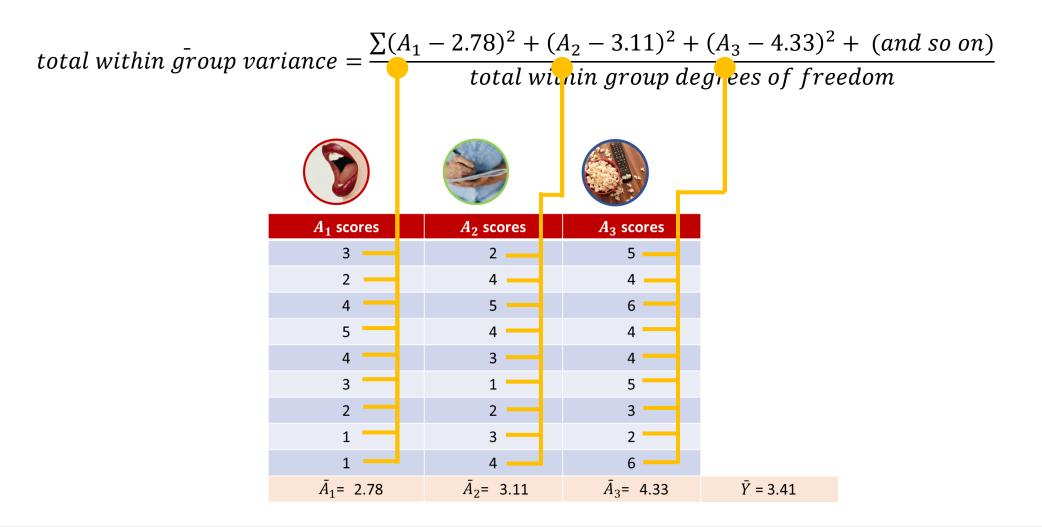




total within group variance = $\frac{\sum (A_1 - \bar{A}_1)^2 + (A_2 - \bar{A}_2)^2 + (A_3 - \bar{A}_3)^2 + (and \ so \ on)}{total \ within \ group \ degrees \ of \ freedom}$







Degrees of freedom

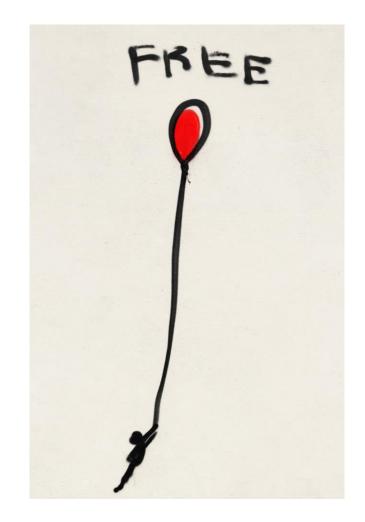


Within-groups degrees of freedom

- For within-groups degrees of freedom, we add up the number of participants for each level – 1
- Mathematically this is expressed as:

$$= (N_{A1} - 1) + (N_{A2} - 1) + (N_{A3} - 1)$$
$$= (9 - 1) + (9 - 1) + (9 - 1)$$

= 24



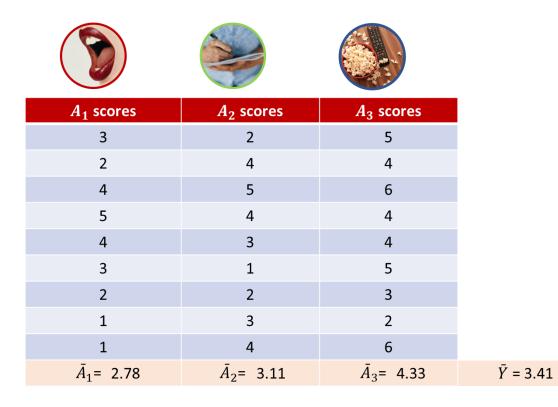


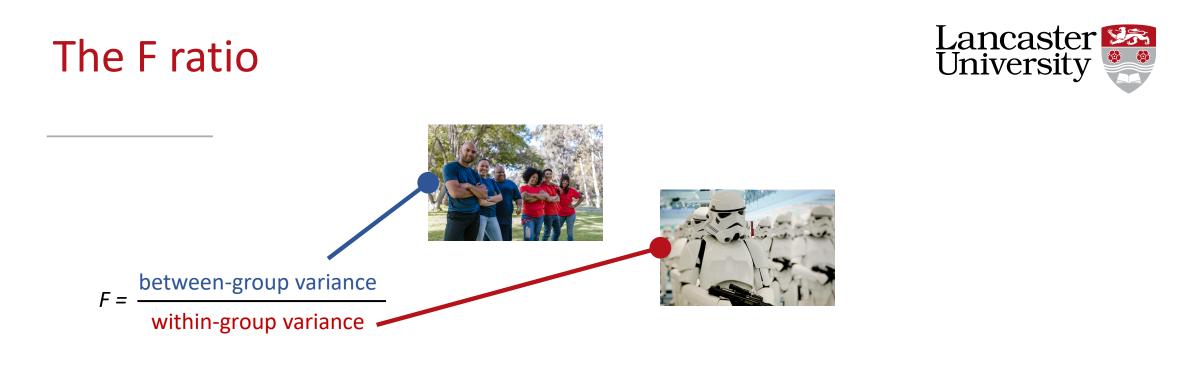
total within group variance =
$$\frac{\sum (A_1 - 2.75)^2 + (A_2 - 3.11)^2 + (A_3 - 4.33)^2}{24}$$

A ₁ scores	A ₂ scores	A ₃ scores			
3	2	5			
2	4	4			
4	5	6			
5	4	4			
4	3	4			
3	1	5			
2	2	3			
1	3	2			
1	4	6			
\bar{A}_1 = 2.78	\bar{A}_2 = 3.11	Ā₃= 4.33			



total within group variance =
$$\frac{42.444}{24}$$
 = 1.769 (with rounding)





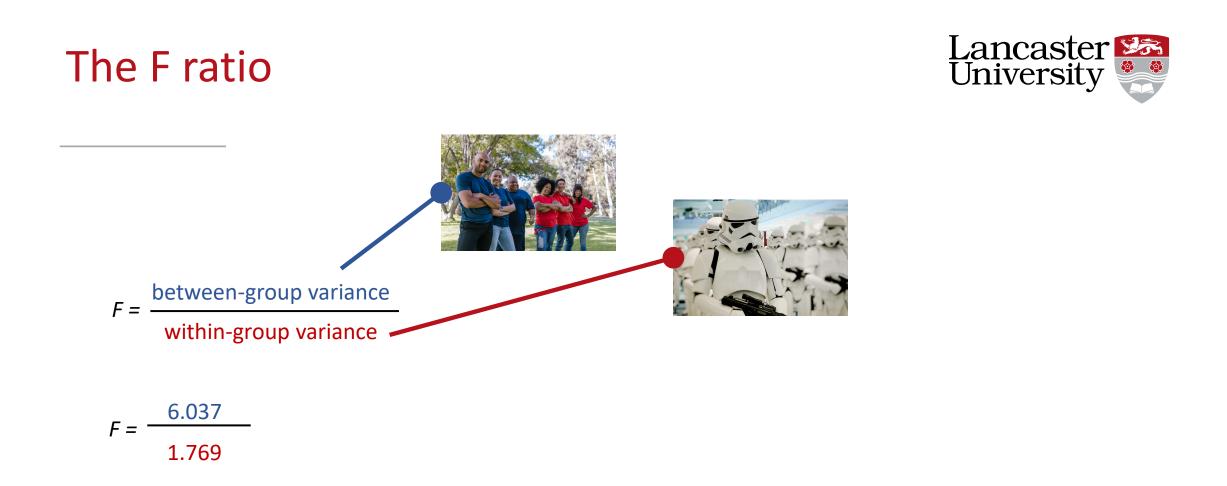
$$F = \frac{6.037}{1.769}$$

F = 3.414



ν_1 ν_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	5.59	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
8	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, Biometrika Tables for Statisticians, Vol. 2 (1972), Table 5, page 178, by permission.



F = 3.414, p = 0.05, A statistically significant test result ($P \le 0.05$)

Lecture 2 – One factor betweenparticipants ANOVA

Review of lecture 2

- What is Analysis of Variance
- What is a one-factor between-participants design
- Sources of variability in data
- Calculated within-group and betweengroup variances
- Degrees of Freedom
- Produced the F-statistic





Check you understand today's lecture – repeat any parts of the lecture you need to.

Don't forget to ask any questions using the Discussion Forum on **Moodle**!

